MHD Instabilities in Solar Eruptions

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1. Intro: Observations & key models

2. Main acceleration
   – Torus instability of flux rope
   – Helical kink instability

3. Further evolution
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   – Kelvin-Helmholtz instability
   – Rayleigh-Taylor instability

4. Recent applications & outlook
1 Introduction

SOHO/EIT & SOHO/LASCO/C2+C3, 2002-Jan-4

- instanton. onset & rapid initial accel. $\Rightarrow$ energy storage & release
- huge expansion of CME ($\sim 10^3$); $v > v_{A,\text{local}}$ $\Rightarrow$ ideal MHD driver
- ex. associated flare $\Rightarrow$ reconnection (non-ideal MHD)
Equations: Magnetohydrodynamics (MHD)

\[
\begin{align*}
\partial_t \rho &= -\nabla \cdot (\rho \mathbf{u}) \\
\rho [\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u}] &= -\nabla p + \mathbf{j} \times \mathbf{B} \quad [ + \rho g + \rho \nu \Delta \mathbf{u}] \\
\partial_t \mathbf{B} &= \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \mathbf{j}) \\
\partial_t \mathbf{u} &= -\nabla \cdot \mathbf{S} \\
\mathbf{j} &= \mu_0^{-1} \nabla \times \mathbf{B} \\
\mathbf{u} &= \rho \mathbf{w} + \frac{\rho}{2} \mathbf{u}^2 + \frac{B^2}{2 \mu_0} ; \quad \mathbf{S} = (\mathbf{u} + p + \frac{B^2}{2 \mu_0}) \mathbf{u} - (\mathbf{u} \cdot \mathbf{B}) \frac{\mathbf{B}}{\mu_0} + \eta \mathbf{j} \times \frac{\mathbf{B}}{\mu_0} \\
p &= (\gamma - 1) \rho \mathbf{w}
\end{align*}
\]

- Ideal MHD approx often OK: \( \eta = 0 \Rightarrow \) Frozen-in field
- Non-ideal MHD (e.g. resistivity; num. diffusion): Reconnection
- Zero-beta approx in active-region corona: \( \beta = p/(B^2/2\mu_0) \sim 10^{-3} \rightarrow 0 \)
The Overall Picture

Filament/prominence eruption
+ Coronal mass ejection (CME)
+ Flare

= large-scale coronal instability

= complex interplay of
  – ideal MHD
  – non-ideal MHD (reconnection)
  – kinetic processes
  – incl. feedback rise–reconnection
Rise Profile: An Evolution in Four Phases

- Quasistatic evolution along equilibrium sequence between events + 3 CME phases
- Activation phase: various scenarios
- Acceleration phase: competing, but currently converging models → This talk
- Propagation phase

Key Open Questions

- Magnetic topology at eruption onset / initial equilibrium
  - arcade of loops
  - flux rope (FR)
  - arcade→FR transition?

- Onset and drive mechanism(s)
  - ideal MHD instability (a.k.a. “loss of equilibrium”)
  - catastrophe (a.k.a. “loss of equilibrium”)
  - magnetic reconnection (non-ideal MHD)
  - coupling (mutual feedback) ideal MHD ↔ reconnection (a.k.a. CME-flare relationship)
Key Open Questions (cont’d)

- Physics of the pre-CME phase (slow-rise/activation phase) / Trigger processes
  - flux cancellation
  - flux emergence
  - onset of reconnection
  - no trigger necessary (“final drop” scenario)

Further Questions

- Origin of different rise profiles
  - fast vs. slow CMEs = different CME types ??
  - exponential vs. power-law rise
  - weak vs. strong rotation

- Helicity outflux & dynamo quenching
Equilibria

Sheared Arcade

- shearing footpoint motions
  \[ \Rightarrow J_{\text{cor}} \neq 0 \]

- no ideal MHD instability or catastrophe known

\[ \Rightarrow \text{reconnection must trigger CME} \]

\[ \rightarrow \text{will produce flux rope} \]


Flux Rope (FR)

- contains current channel:
  \[ J_{\text{cor}} \neq 0 \]

- various forms of MHD instability:
  - torus instability
  - helical kink instability

Bateman 1978 “MHD Instabilities”
CME Models: Onset & Acceleration

Tether Cutting (1992): “runaway” reconnection inside magnetic arcade

Magnetic Breakout (1999): “breakout” reconnection above triple magn. arcade

Arcade models:
Flux rope forms after CME onset

Flux Rope Catastrophe (1991): end point in equilibrium sequence & jump

Flux Rope Instability (2003): ideal MHD instability (torus & kink instabilities)

Flux rope models:
Flux rope exists prior to CME onset
Relevant Instabilities

Lateral kink / torus instability

Helical \((m = 1)\) kink

Plasmoid instab.

Kelvin-Helmholtz inst.

Rayleigh-Taylor inst.

C.Shen+ 11, 13; Z.Mei+ 12; L.Ni+ 12

Foullon et al. 2011, 2013

Innes et al. 2012

Gerrard et al. 2001, A&A
2 CME Onset & Main Acceleration
– A Flux Rope Instability

Assumptions

- Single flux rope topology
- This topology preserved \(\rightarrow\) ideal MHD
- Seek to describe this as an instability
Force Balance in Flux Rope Equilibrium

Flux rope above photosphere induces current in photosphere
Equivalent to image current
⇒ Repelling (upward) force

Force-free equilibrium requires opposite force
Provided by $J \times B$ with *external poloidal field*
(often loosely called “overlying field”)

Kuperus & Raadu 1974 A&A
van Tend & Kuperus 1978 SP
Instability of Flux Rope Equilibrium

- The transition between stability and instability must be controlled by the external poloidal field $B_{ep}$ (the “overlying arcade”)

- If $B_{ep}(h)$ falls off too rapidly with height, then upward displacements will be unstable

- 2D: $n = -\frac{d \ln B_{ep}}{d \ln h} > n_{cr} = 1$ (van Tend & Kuperus 1978)

- 3D: $n > n_{cr} \approx 3/2$ “Torus Instability” (a lateral kink) (Bateman 1978; Kliem & Török 2006 PRL, Démoulin & Aulanier 2010 ApJ)

- High twist, $\Phi = 2\pi N > \Phi_{cr} = (2.5 \ldots \sim 5)\pi$, additionally triggers the helical ($m = 1$) kink mode (Sakurai 1976 PASJ; Hood & Priest 1981 GAFD; Fan & Gibson 2003 ApJL; Török et al. 2004 A&A Lett)

\[1\] THE key paper in CME theory
2.1 Torus Instability (TI)

The Expansion Instability of a Current Ring

Instability condition:

\[ n = - \frac{\frac{\text{d} \ln B_{\text{ex}}}{\text{d} \ln R}}{n_{\text{cr}}} > n_{\text{cr}} = \frac{3}{2} \]

Bateman, MHD Instabilities, 1978

The TI is a lateral kink, uniformly distributed over the ring
**TI Description**

\[ B_{\text{ex}}(R) = \hat{B} R^{-n} \quad \text{(for } R \geq R_0) \]

Consider freely expanding ring, neglect flux pileup, \( \nabla p \), gravity

**Ideal MHD:** conservation of flux

\[ \Rightarrow \quad I = I(R, n, I_0, R_0, a_0, l_i) \quad \text{(decreases during expansion!)} \]

\[ \rho_m \frac{d^2R}{dt^2} = \frac{I^2}{4\pi^2a^2R^2}(L + \mu_0R/2) - \frac{IB_{\text{ex}}(R)}{\pi a^2} \]

\[ L = \mu_0R \left( \ln \left( \frac{8R}{a} \right) - 2 + \frac{l_i}{2} \right) \]; assume self-similar expansion: \[ \frac{a}{R} = \frac{a_0}{R_0} \]
Torus Instability Simulation & “Granddaddy”

\[ n \sim 3 > n_{cr} \approx \frac{3}{2} \]

\[ (\Phi = 2.5\pi < \Phi_{cr}) \]
TI – Observational and Computational Support

Simulations

Kliem & Török 2007; Aulanier et al. 2010; Fan 2010

Potential field

Y.Liu 2008;
Y.Guo et al. 2010;
Y.Shen et al. 2012;
Kumar et al. 2012;
Filippov 2013

NLFFF

X.Cheng+ 2011a, 2013b; C.Jiang+ 2013; see also Kliem+ 2013

Counter indications (?)

Properties of the TI

- Threshold is formally independent of current
  \(\rightarrow\) facilitates energy storage

- Stable in near field and unstable in far field of bipole/quadrupole
  \(\rightarrow\) consistent with (slow) rise before eruption
  \(\rightarrow\) may need external agent to lift flux into unstable height range

- Flux rope rises into the strongly unstable domain
  \(\rightarrow\) vigorous evolution

- Never saturates/ends completely (unlike helical kink instability)
  \(\rightarrow\) operates at any distance from Sun

- CME velocity depends on Alfvén speed (field strength) and decay index (height profile) of external field
  \(\rightarrow\) consistent with weak correlation of CME vs. flare magnitude
The TI and Fast & Slow CMEs

\( n \gtrsim 2 \) (active regions):
- fast expansion
- exponential-to-linear evolution

\( n \gtrsim n_{cr} \) (quiet Sun):
- slow expansion
- nearly constant acceleration

Kliem & Török 2006, PRL 96, 255002
Török & Kliem 2007, AN 328, 743
Y.Xu et al. 2012
MacQueen & Fisher 1983
Simulation of Fast & Slow CMEs

Fast CMEs from AR, slow CMEs from QS, fastest CMEs from δ spots
Exponential vs. Power-law Rise Profiles

- expon. $\rightarrow$ power-law rise for perturb. vel. $v_0 \sim 0.03 \, V_A$

2.2 The Helical \((m = 1)\) Kink Instability (KI)

- Ideal MHD instability of a current channel
- Helical Kink converts twist \(\rightarrow\) writhe
- **Threshold twist:** \(\Phi_{\text{cr}} = 2\pi N_{\text{cr}} \sim (2.5 \ldots)\pi\)
  
  - \(\Phi \propto B_\phi \propto j \propto W_{\text{mag, free}}\)


• KI saturates quickly; appears to act more frequently in confined ("failed") eruptions than in CMEs
Modelling a Confined Filament Eruption as a Helical Kink

TRACE Fe XII 195 Å (~ 1.5 MK)

\( \Phi = 5\pi > \Phi_{cr} \)


Török & Kliem 2005 ApJL
KI in Confined Eruption (cont’d)
KI & Current Sheets; Sigmoid; Flux Rope Reformation

- helical current sheet (CS)
- vertical ("flare") CS → sigmoid

\( \Phi \sim 3-5\pi \); Kliem et al. 2004 A&A

\( \Phi \geq 6\pi \); Kliem et al. 2010 SP
Role of the KI in CMEs

• May provide initial lift into torus-unstable height range
• Good explanation for confined eruptions
• Cannot explain huge expansions of CMEs by $\geq 10^3$
• Cannot explain eruptions with little or no apparent helical shape
• Helical shape is not a proof of KI occurrence!

3 Instabilities in CME Evolution

3.1 Plasmoid Instability

Current sheet breakup into plasmoids
- yields high reconnection rates
- transitions to turbulent reconnection
→ observed high current sheet widths
- explains some radio burst fine structures
- may explain recurrent jets
- enables diagnostics of reconnection

- BUT is NOT the origin of the eruptions
3.2 Kelvin-Helmholtz Instability

Good agreement w. linear KH theory

\[ \mathbf{v}_g = \frac{n_1 \mathbf{V}_1 + n_2 \mathbf{V}_2}{n_1 + n_2} \]
3.3 Rayleigh-Taylor Instability

\[ \lambda_{cr} = \frac{B^2}{g(\rho_h - \rho_l)} \Rightarrow V_A \approx 47 \text{ km s}^{-1} \]
4 Some Further Applications (Outlook)

4.1 CME-Flare Relationship and Field Topology


Maricic et al. 2007 SP

Kliem et al. 2013, in prep.

CME onset before flare in ~ 25% of events
4.2 CME Cavity Formation

Expansion outside flux rope due to "reverse pinch effect"

Kliem et al. 2013, in prep.
4.3 Towards Data-driven CME Simulations

“Flux imbalance” onset condition consistent with TI onset condition
5 Summary

- **Flux rope instability models** have made considerable progress:
  - provide *quantitative* onset conditions:
    \[ n > n_{cr} \sim 3/2; \Phi > \Phi_{cr} \sim 3.5\pi, \]
    - based on the *coronal* field
  - put fast and slow eruptions in single physical framework
  - reproduce specific properties in many events (velocity; rotation; inclination; shape; ...)

- \( \Rightarrow \) **Torus instability/Catastrophe** is the main driver of CMEs

- The **helical kink instability** appears to provide the initial kick in some events

- Still a long way to reliable forecasts
  that are based on the *photospheric* field